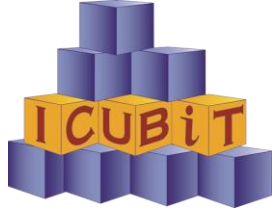
IMPROVING CURRICULUM USE FOR BETTER TEACHING

Examining Mathematics Curriculum Materials from the Perspective of Teacher Use

NCTM Research Presession

April 12, 2011

About the Project



- NSF Study: Improving Curriculum Use for Better Teaching (ICUBiT)
- PDC: Individual teacher's ability to perceive and mobilize existing curricular resources in order to design instruction (Brown, 2009)
- Goal:
  - Identify the components of PDC that support curriculum use
  - Develop tools for measuring it

Curriculum Analysis

- Pedagogical Design Capacity

Curriculum Design

- Questions:

  What demands does the curriculum place on teachers?

  What supports does the curriculum provide the teacher?

Five Curriculum Programs

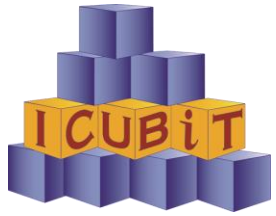


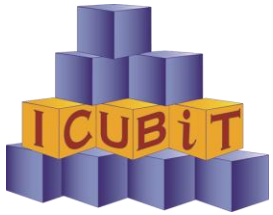
Abb.	Curriculum Title	Developers	Current Publisher
EM	*Everyday Mathematics* (3rd Edition)	University of Chicago Mathematics Project	Wright Group/ McGraw-Hill
INV	*Investigations in Numbers, Data, and Space* (2nd Edition)	TERC	Pearson
SF	Scott Foresman Mathematics	Scott Foresman/Pearson	Pearson
SM	Primary Mathematics (Standards Editions)	Singapore Ministry of Education	Marshall Cavendish International
TB	Math Trailblazers (3rd Edition)	TIMS at University of Illinois at Chicago	Kendall Hunt

Analytical Framework

- Model Lesson

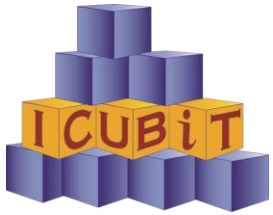
- Voice of the text

Analytical Framework



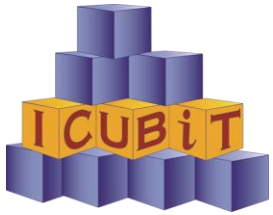
- Model Lesson
  - Researcher's model of the author-intended curriculum (lesson level) (Brown, 2008)
    - Mathematical Emphasis
    - Cognitive Demand
    - Key Instructional Representations
    - Instructional Approach (Teacher and student roles)
- Voice of the text

Analytical Framework



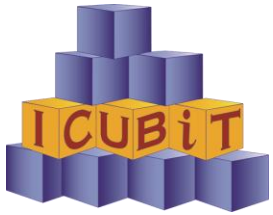
- Model Lesson (*Imagined Lesson)*
  - Researcher's model of the author-intended curriculum (lesson level) (Brown, 2008)
    - Mathematical Emphasis
    - Cognitive Demand
    - Key Instructional Representations
    - Instructional Approach (Teacher and student roles)
- Voice of the text
  - How the text communicates with the teacher
  - What it communicates about
  - How the text positions the teacher

Analytical Framework



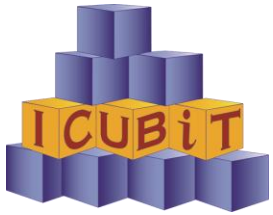
- Model Lesson (*Imagined Lesson)*
  - Researcher's model of the author-intended curriculum (lesson level) (Brown, 2008)
    - Mathematical Emphasis
    - Cognitive Demand
    - Key Instructional Representations
    - Instructional Approach (Teacher and student roles)
- Voice of the text
  - How the text communicates with the teacher
  - What it communicates about
  - How the text positions the teacher

Methods



- Focus on numbers, operations, Algebra
- Grades 3-5
- Reviewed entire curriculum to understand structure, key features, and emphasis
- Systematically analyzed 3 lessons from each grade (randomly selected)
- Coded for cognitive demand, teacher and student roles, types of communication with the teacher
- Cross-curricular analysis

Cognitive Demand



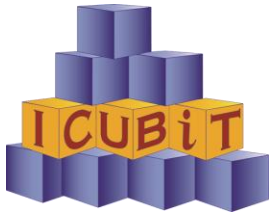
+Memorization (Mem)

+Procedures Without Connections (PWOC)

+Procedures with Connections (PWC)

+Doing Mathematics (DM)

Teacher's Role



+Showing, telling, directing

+Guiding

+Facilitating

+Orchestrating

Voice of the Text

1. Directing Action, providing information
2. Explaining rationale
3. Anticipating student thinking
4. Explaining the math
5. Supporting teacher decision making

Voice of the Text

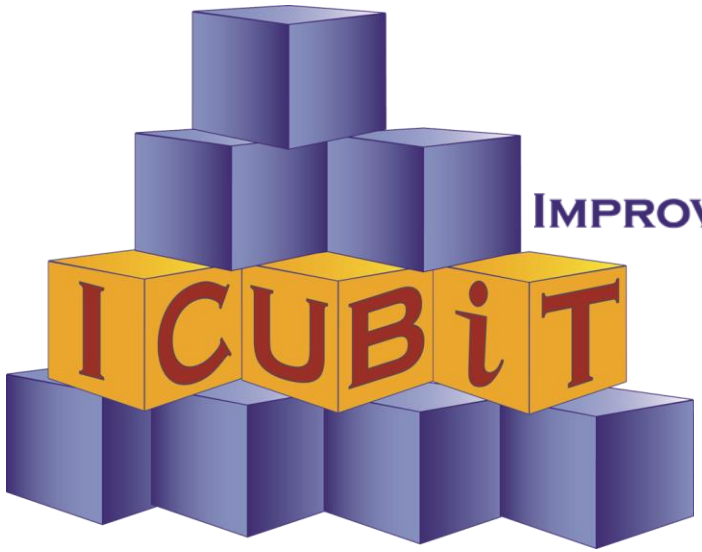
Type of Support	Examples
Directing Action (providing Information)	Guide students through the subtraction algorithm step-by-step. (SM) Ask children to share other strategies they might use to solve the number story, as you make notes on the board. (EM)
Explaining Rationale	Review the unit box as a way of establishing a real-world context for numbers. (EM) Making representations for these different situations helps students see the actions in each type of problem and how they can use addition and subtraction to solve them. (INV)

Voice of the Text

Type of Support	Examples
Anticipating Student Thinking	Students should understand that the properties justify the steps shown in the three students' papers. (SF) In question 2, a student who understands place value should respond with 40 or 4 tens. (TB)
Explaining Math	Properties of whole numbers explain why you can choose which numbers to multiply first. (SF) The U.S. algorithm for subtraction, sometimes called "borrowing" or the regrouping algorithm , is a procedures that was devised for compactness and efficiency. (INV)
Supporting Teacher Decision Making	A brief review of this lesson's materials may suffice for your class (TB) If you wish, ask children to write a complete sentence to answer the problem. (EM)

Presentation Structure

- Background and development
- Description of resources
- Model lesson
  - Structure
  - Cognitive demand
  - Teacher's and student's role
- Types and nature of guidance
- Summary of demands and assumptions



IMPROVING CURRICULUM USE FOR BETTER TEACHING

Analysis of Everyday Mathematics

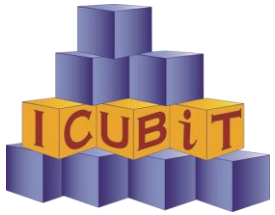
Shari Lewis

Western Michigan University

Joshua Taton

University of Pennsylvania

Everyday Mathematics

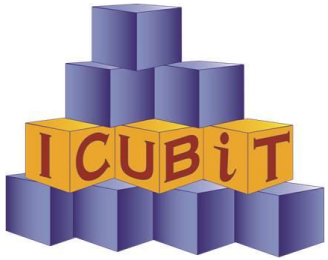


- Developed by the University of Chicago School Mathematics Project
  - NSF Instructional Materials Development Projects
  - 3rd Edition was used for this analysis
- A "spiraled" program – Teacher's Lesson Guide
- Student Materials
  - Student Math Journal – consumable workbook
  - Student Reference Book
  - Home Links – consumable workbook

Everyday Mathematics

- Teacher Materials
  - Teacher's Lesson Guide
  - Teacher's Reference Manual
  - Assessment Handbook
  - Differentiation Handbook
  - Home Connection Handbook
  - Minute Math

Everyday Mathematics – Teacher's Guide



Contains:

- Unit Organizers (not analyzed)
  - Overview
  - Links to the past and future
  - Ongoing & Periodic Assessment
  - Materials List
  - Unit Project

- Lessons

Getting Started

Mental Math and Reflexes

Write decimals on the board and ask students to read them. Suggestions:

0.5	34.12	0.984
0.76	9.03	0.733
0.14	465.81	0.804

Math Message

Solve Problem 1 on journal page 82.

Study Link 4·2 Follow-Up

Have students share examples of decimals they brought from home. Discuss their meanings and values. Use such language as, *The label on a package of chicken reads "2.89 pounds." 2.89 pounds is between 2 and 3 pounds. It is almost 3 pounds.* Encourage students to continue bringing examples of decimals to display in a Decimals All Around Museum. See the optional ELL Support activity in Part 3 for details.

1 Teaching the Lesson

Math Message Follow-Up

WHOLE-CLASS DISCUSSION

(*Math Journal 1*, p. 82)

Discuss ways to show that $0.3 > 0.15$. Be sure to include the following two methods:

- Model **decimals** with base-10 blocks. If a flat is ONE, then 0.3 is $\frac{3}{10}$ of the flat, or 3 longs, and 0.15 is $\frac{15}{100}$ of the flat, or 15 cubes. Because 3 longs are more than 15 cubes, $0.3 > 0.15$.
- Rename one of the decimals so that both decimals have the same number of digits to the right of the decimal point. Do so by appending zeros to the decimal having fewer digits after the decimal point. In this problem, show that $0.3 = 0.30$ by trading 3 longs for 30 cubes. Because 30 cubes are more than 15 cubes, $0.30 > 0.15$. Therefore, $0.3 > 0.15$.

Have students use base-10 blocks to complete Problem 2 on journal page 82.

Ongoing Assessment: Informing Instruction

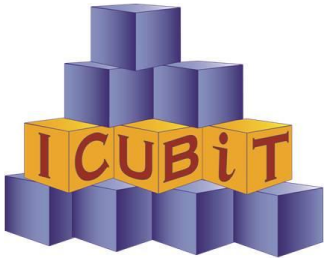
Watch for students who think 0.3 is less than 0.15 because 3 is less than 15. Modeling the problems with base-10 blocks and then trading longs for cubes can help students understand why zeros can be appended to a decimal without changing its value.

Writing a zero at the end of a decimal corresponds to thinking about the number in terms of the next smaller place. For example, 30 hundredths, 0.30, or 30 cubes is greater than 15 hundredths, 0.15, or 15 cubes. Note how this differs from the situation with whole numbers: With whole numbers, the number with more digits is always greater.

Student Page

Math Journal 1, p. 82

Model Lesson – Common Structure



What does the model lesson include?

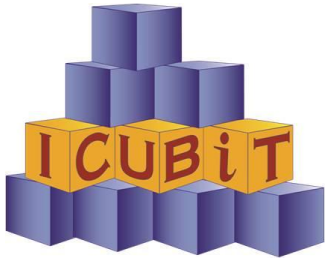
- Getting Started
- Teaching the Lesson
- Ongoing Learning & Practice
- Differentiation Options

Model Lesson – Cognitive Demand

18 Tasks were analyzed

Mem	PWOC	PWC	DM
4 22%	4 22%	9 50%	1 6%

Model Lessons - Roles



Teachers' Role	Students' Role
• Facilitate discussions by using curriculum provided prompts or posing suggested problems. • Guiding Role, less didactic than telling but still primary shaper of classroom interactions.	• Discuss mathematics with teacher • Discuss mathematics with peers • Transition from intuition to concrete operations and eventually to abstract

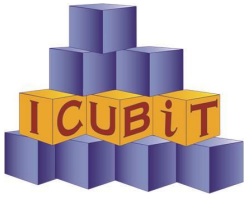
Guidance for Teachers

Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
3, U1, 1.8	4	95	80 (84.2%)	6 (6.3%)	0 (0%)	9 (9.5%)	9 (9.5%)
3, U2, 2.1	6	133	103 (77.4%)	12 (9.0%)	10 (7.5%)	8 (6.0%)	7 (5.3%)
3, U4, 4.1	6	126	107 (84.9%)	7 (5.6%)	10 (7.9%)	2 (1.6%)	15 (11.9%)
4, U3, 3.1	5	88	68 (77.3%)	10 (11.4%)	8 (9.1%)	2 (2.3%)	8 (9.1%)
4, U3, 3.2	6	129	113 (87.6%)	8 (6.2%)	2 (1.6%)	6 (4.7%)	9 (7.0%)
4, U5, 5.5	6	116	86 (74.1%)	10 (8.6%)	15 (12.9%)	5 (4.3%)	9 (7.8%)
5, U1, 1.3	5	101	75 (74.3%)	14 (13.9%)	9 (8.9%)	3 (3.0%)	8 (7.9%)
5, U2, 2.4	6	148	101 (68.2%)	8 (5.4%)	11 (7.4%)	28 (18.9%)	5 (3.4%)
5, U4, 4.1	6	112	89 (79.5%)	9 (8.0%)	14 (12.5%)	0 (0%)	6 (5.4%)
Mean Median Range			91.3 89.0 68-113	9.3 9 6-14	8.8 10 0-15	7 5 0-28	8.4 8 5-15

Guidance for Teachers

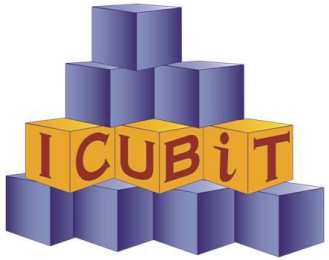
Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
3, U1, 1.8	4	95	80 (84.2%)	6 (6.3%)	0 (0%)	9 (9.5%)	9 (9.5%)
3, U2, 2.1	6	133	103 (77.4%)	12 (9.0%)	10 (7.5%)	8 (6.0%)	7 (5.3%)
3, U4, 4.1	6	126	107 (84.9%)	7 (5.6%)	10 (7.9%)	2 (1.6%)	15 (11.9%)
4, U3, 3.1	5	88	68 (77.3%)	10 (11.4%)	8 (9.1%)	2 (2.3%)	8 (9.1%)
4, U3, 3.2	6	129	113 (87.6%)	8 (6.2%)	2 (1.6%)	6 (4.7%)	9 (7.0%)
4, U5, 5.5	6	116	86 (74.1%)	10 (8.6%)	15 (12.9%)	5 (4.3%)	9 (7.8%)
5, U1, 1.3	5	101	75 (74.3%)	14 (13.9%)	9 (8.9%)	3 (3.0%)	8 (7.9%)
5, U2, 2.4	6	148	101 (68.2%)	8 (5.4%)	11 (7.4%)	28 (18.9%)	5 (3.4%)
5, U4, 4.1	6	112	89 (79.5%)	9 (8.0%)	14 (12.5%)	0 (0%)	6 (5.4%)
Mean Median Range			78.6% 77.4% 68.2-87.6%	8.3% 8.0% 5.4- 13.9%	7.5% 7.9% 0-12.9%	5.6% 4.3% 0-18.9%	7.5% 7.8% 3.4-11.9%

Guidance for Teachers

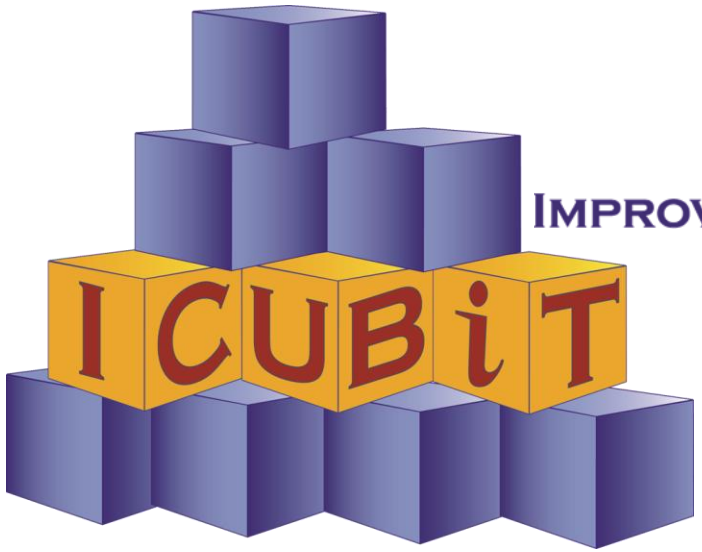


Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
3, U1, 1.8	4	95	80 (84.2%)	6 (6.3%)	0 (0%)	9 (9.5%)	9 (9.5%)
3, U2, 2.1	6	133	103 (77.4%)	12 (9.0%)	10 (7.5%)	8 (6.0%)	7 (5.3%)
3, U4, 4.1	6	126	107 (84.9%)	7 (5.6%)	10 (7.9%)	2 (1.6%)	15 (11.9%)
4, U3, 3.1	5	88	68 (77.3%)	10 (11.4%)	8 (9.1%)	2 (2.3%)	8 (9.1%)
4, U3, 3.2	6	129	113 (87.6%)	8 (6.2%)	2 (1.6%)	6 (4.7%)	9 (7.0%)
4, U5, 5.5	6	116	86 (74.1%)	10 (8.6%)	15 (12.9%)	5 (4.3%)	9 (7.8%)
5, U1, 1.3	5	101	75 (74.3%)	14 (13.9%)	9 (8.9%)	3 (3.0%)	8 (7.9%)
5, U2, 2.4	6	148	101 (68.2%)	8 (5.4%)	11 (7.4%)	28 (18.9%)	5 (3.4%)
5, U4, 4.1	6	112	89 (79.5%)	9 (8.0%)	14 (12.5%)	0 (0%)	6 (5.4%)
Mean Median Range			78.6% 77.4% 68.2-87.6%	8.3% 8.0% 5.4- 13.9%	7.5% 7.9% 0-12.9%	5.6% 4.3% 0-18.9%	7.5% 7.8% 3.4-11.9%

Demands and Assumptions



Demands on Teacher	Assumed Knowledge
• Use of manipulatives • Connecting Mathematics to real-world contexts to • Enact a wide variety of activities • Familiarity with mathematics vocabulary and multiple methods • Recognize the importance of games in this curriculum	• To use manipulatives to enhance lesson and not distract from key concepts • Make mathematical concepts relevant • Familiar with multiple algorithms/procedures (or will use the Teacher's Resource Manual)



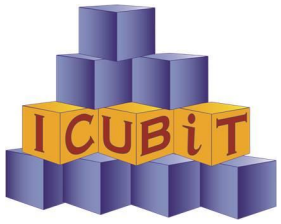
IMPROVING CURRICULUM USE FOR BETTER TEACHING

Analysis of *Investigations in Number, Data and Space*

Napthalin A. Atanga

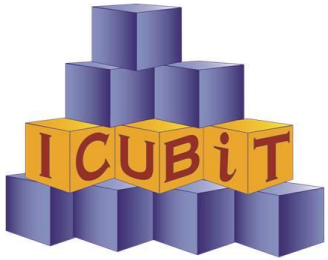
Western Michigan University

Investigations



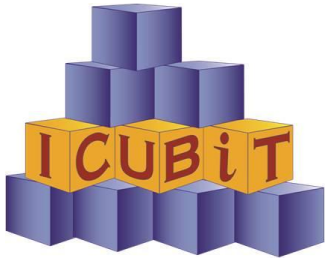
- **Developed by Educational Researchers at TERC and the *2nd Ed* was published in 2008.**
- **Organization:** It has 9 units per grade; 2-4 investigations in each unit; 4-9 sessions.
- **Material for students:** consumables pages such as recording sheets, homework, and practice sheets.
  - Student math handbook (Math word and idea pages, and game directions)

Investigations



- Materials for Teachers
  - Teacher's guide
  - Implementation guide
  - Resource binder
    - Resource masters (available on CD)
    - CD containing student software

Investigations - Teacher's Guide



- Each unit includes:
  - Overview of the unit
  - Mathematics in the unit
  - Assessment (ongoing, writing opportunities, portfolio opportunities)
  - Algebra connections
  - Classroom routines and ten-minute math
  - Practice and review
  - Differentiation
  - Planner for each investigation
  - End of unit assessment
  - Assessing the bench mark

ACTIVITY

1 Introducing Place Value: Stickers and Cubes

20 MIN CLASS PAIRS

This activity uses stickers and cubes to help students understand place value. In addition, the activity introduces the Ten-Minute Math activity *Practicing Place Value*, which students will continue to do throughout this unit and at other times during the school year.

Distribute 100 cubes in towers of 10 to each pair of students.

For the next few weeks, we will be working on addition and subtraction. We'll use some different tools, such as stickers, 100 charts, and number lines.

Most of you probably remember Sticker Station from Grade 2. Sticker Station is a very popular store that sells all kinds of stickers. To keep things organized, and to make buying stickers easier, Sticker Station sells stickers in different ways. You can buy individual stickers called "singles," or you can buy strips of stickers.

On the overhead, show a few transparent singles and strips that you have prepared from Stickers: Strips and Singles (T2).

What do you notice about the way these stickers are organized? How many stickers do you get on a strip? How many singles is that equal to?

Establish that there are 10 stickers on a strip and that 10 singles are equivalent to one strip of 10. Then use the transparent stickers to pose a few problems.

Display four strips of 10 and 6 singles.

Professional Development

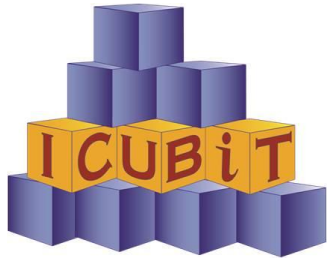
- **Teacher Note:** Place Value, p. 143
- **Teacher Note:** Stickers: A Context for Place Value, p. 145
- **Part 4: Ten-Minute Math and Classroom Routines:** *Implementing Investigations in Grade 3:* Practicing Place Value

Math Note

- **Math Tools** Number lines and 100 charts are useful tools for understanding and representing the operations of addition and subtraction. Students worked with both of these tools in Grade 2. Throughout this unit and for the remainder of the year, post a class number line and 100 chart in visible places in the classroom.

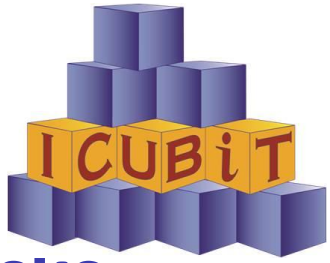
▲ Transparencies, T2

Model Lesson –Common Structure



- **Each session consists of a combination of:**
  - Ten-Minute Math
  - Task
  - Discussion
  - Math Workshop
  - Ongoing Assessment
  - Differentiation
  - Session Follow-Up

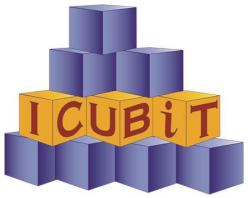
Model Lesson –Cognitive Demand



9 lessons were analyzed with 11 main tasks

Mem	PWOC	PWC	DM
0	0	5	6
0%	0%	45%	55%

Model Lessons - Roles



Teachers' Role

- Assign tasks to students
- Monitor students at work
- Assess students' progress
- Asked suggested questions
- Probe students thinking
- Encourage sharing of ideas and strategies

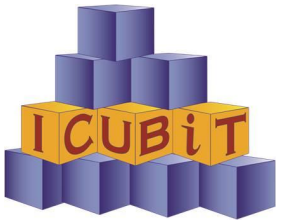
Students' Role

- Engage with the task,
- Observe patterns,
- Solve non-routine problems
- Invent solution strategies,
- Share their thinking,
- Collaborate with others
- Interpret and use visual models

Guidance for Teachers

Lesson	Pages per lesson	Sentences/ Phrases Per Lesson	Directing action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
3, U1, 1.1	10	154	118 (76.7%)	19 (12.3%)	12 (7.8%)	0 (0%)	5 (3.2%)
3, U1, 1.2	8	133	104 (78.2%)	7 (5.3%)	18 (13.5%)	2 (1.5%)	2 (1.5%)
3, U1, 2.2	7	113	92 (81.4%)	3 (2.7%)	10 (8.8%)	6 (5.3%)	2 (1.8%)
4, U5, 3.3	5	73	55 (75.3%)	3 (4.1%)	10 (13.7%)	1 (1.4%)	4 (5.5%)
4, U5; 4.2	6	90	56 (62.2%)	8 (8.9%)	21 (23.3%)	5 (5.6%)	0 (0%)
4, U9; 2.7	6	93	72 (77.4%)	5 (5.4%)	15 (16.1%)	1 (1.1%)	0 (0%)
5, U1; 2.1	7	138	106 (76.8%)	13 (9.4%)	15 (10.9%)	2 (1.45%)	2 (1.45%)
5, U3, 2.4	6	102	63 (61.8%)	12 (11.8%)	11 (10.8%)	11 (10.8%)	5 (4.9%)
5, U8, 2.4	8	137	108 (78.8%)	2 (1.5%)	14 (10.2%)	11 (8%)	2 (1.5%)
		Mean	74.3%	6.8%	12.8%	3.9%	2.2%
		Median	76.8%	5.4%	10.9%	1.5%	1.5%
		Range	61.8-81.4%	1.5-12.3%	7.8-23.3%	0.0-10.8%	0.0-5.5%

Guidance from Designers



ACTIVITY 1

Introducing Place Value: Stickers and Cubes

20 MIN CLASS PAIRS

This activity uses stickers and cubes to help students understand place value. In addition, the activity introduces the Ten-Minute Math activity *Practicing Place Value,* which students will continue to do throughout this unit and at other times during the school year.

Distribute 100 cubes in towers of 10 to each pair of students.

For the next few weeks, we will be working on addition and subtraction. We'll use some different tools, such as stickers, 100 charts, and number lines.

Most of you probably remember Sticker Station from Grade 2. Sticker Station is a very popular store that sells all kinds of stickers. To keep things organized, and to make buying stickers easier, Sticker Station sells stickers in different ways. You can buy individual stickers called "singles," or you can buy strips of stickers.

On the overhead, show a few transparent singles and strips that you have prepared from Stickers: Strips and Singles (T2).

What do you notice about the way these stickers are organized? How many stickers do you get on a strip? How many singles is that equal to?

Establish that there are 10 stickers on a strip and that 10 singles are equivalent to one strip of 10. Then use the transparent stickers to pose a few problems.

Display four strips of 10 and 6 singles.

Professional Development

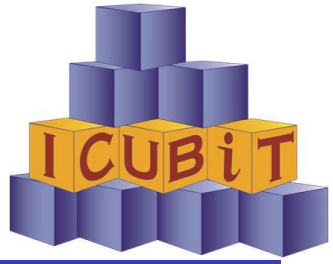
- **Teacher Note:** Place Value, p. 143
- **Teacher Note:** Stickers: A Context for Place Value, p. 145
- **Part 4: Ten-Minute Math and Classroom Routines:** *Implementing Investigations in Grade 3:* Practicing Place Value

Math Note

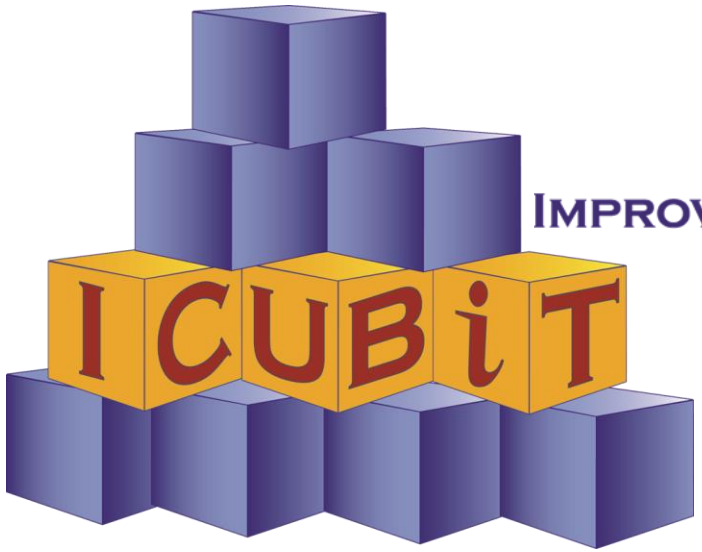
Math Tools Number lines and 100 charts are useful tools for understanding and representing the operations of addition and subtraction. Students worked with both of these tools in Grade 2. Throughout this unit and for the remainder of the year, post a class number line and 100 chart in visible places in the classroom.

Transparencies, T2

Demands and Assumptions



Demands on Teacher	Assumed Knowledge
• High Cognitive Demand • Follow the Curriculum • Anticipation of Students' thinking • Make-decision	• Subject Matter Knowledge • Pedagogical Content Knowledge



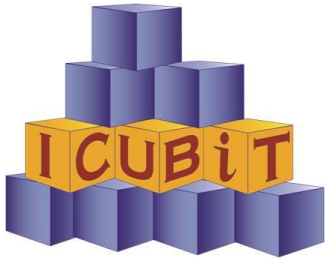
IMPROVING CURRICULUM USE FOR BETTER TEACHING

Analysis of Scott Foresman Mathematics

Nina Hoe

University of Pennsylvania

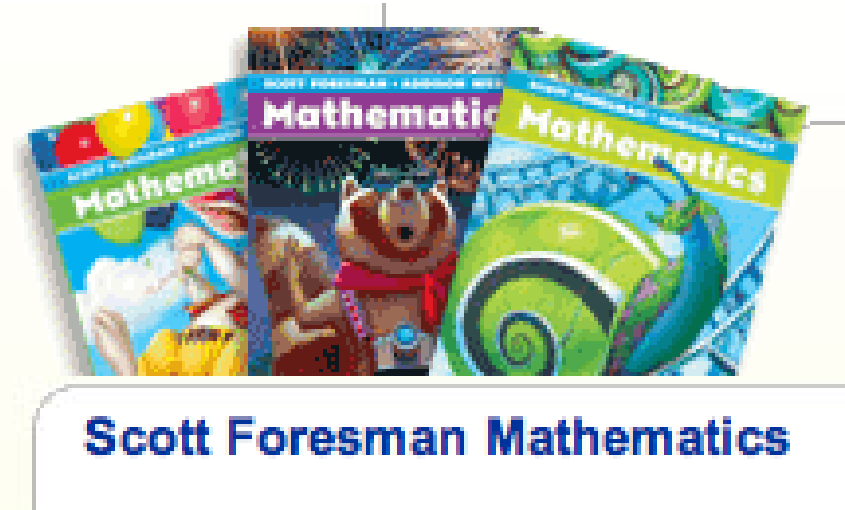
Scott Foresman Mathematics



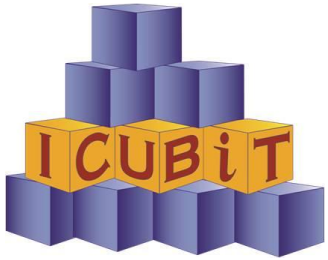
- Commercially Developed
- Owned and Published by Pearson
  - 2008 edition was used for this analysis
- Organized into 12 chapters, each containing 10 – 16 lessons; content and structure is similar across grades 3 - 5
- Student Materials
  - Student Textbook
  - Homework Workbook

Scott Foresman Mathematics

- Teacher Materials
  - Teacher's Edition
  - Teacher Resource Package
    - Practice Masters/Workbook
    - Reteaching Masters/Workbook
    - Enrichment Masters/Workbook
    - Problem Solving Masters/Workbook
    - Homework Workbook Answer Key
    - Test Prep Masters/Workbook and Answer Teaching Tool Masters
    - Assessment Sourcebook
    - Every Student Learns
    - Spiral Review
    - Home-School Connection
    - Chapter File Folders
    - Digital Learning CD-ROM



Scott Foresman Mathematics – Teachers Guide

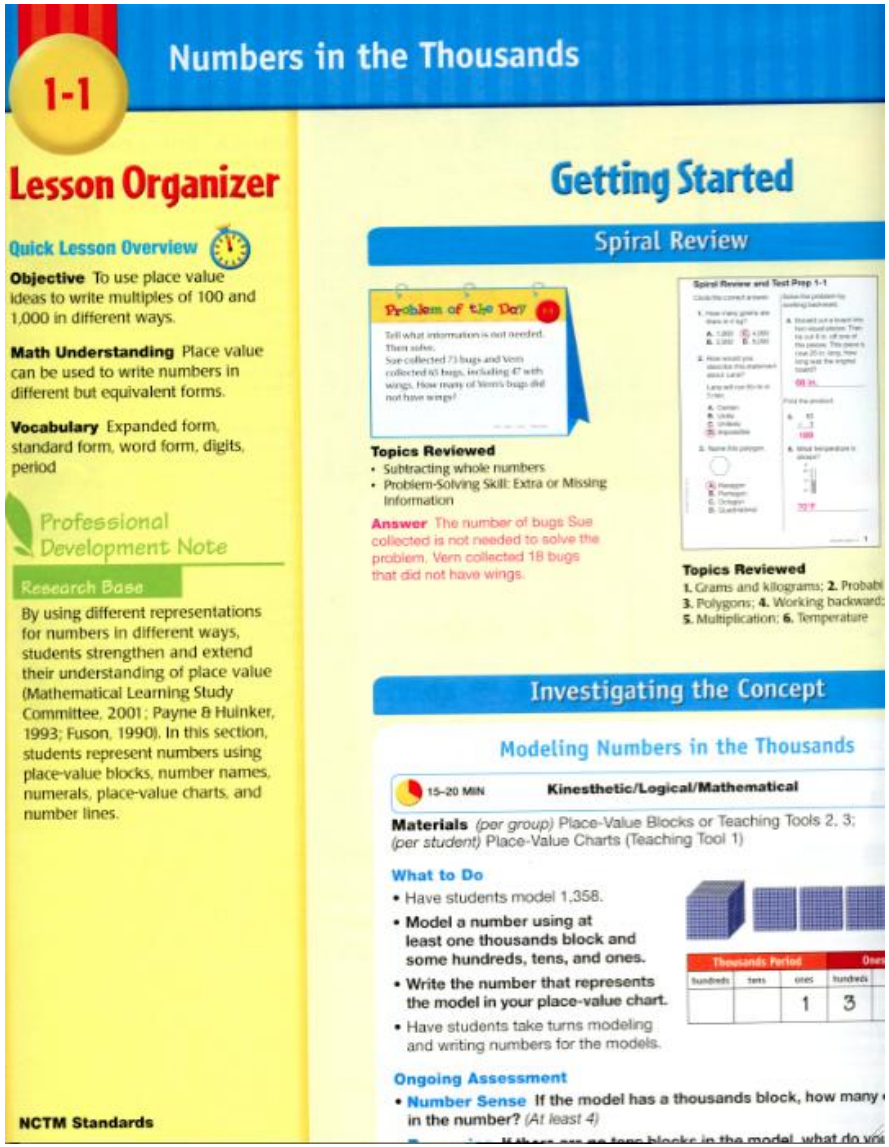


Contains:

- Chapter Organizers (not analyzed)
  - Problem of the Day
  - Table of Contents
  - Lesson Planner
  - Assessment, Intervention, Test Prep
  - Skills Trace
  - Math Background and Teaching Tips

- Lessons: guidance for teaching each lesson

Instructional Page



1-1 Numbers in the Thousands

Lesson Organizer

Quick Lesson Overview

Objective To use place value ideas to write multiples of 100 and 1,000 in different ways.

Math Understanding Place value can be used to write numbers in different but equivalent forms.

Vocabulary Expanded form, standard form, word form, digits, period

Professional Development Note

Research Base

By using different representations for numbers in different ways, students strengthen and extend their understanding of place value (Mathematical Learning Study Committee, 2001; Payne & Huinker, 1993; Fuson, 1990). In this section, students represent numbers using place-value blocks, number names, numerals, place-value charts, and number lines.

NCTM Standards

Getting Started

Spiral Review

Problem of the Day

Topics Reviewed
- Subtracting whole numbers
- Problem-Solving Skill: Extra or Missing Information

Answer The number of bugs Sue collected is not needed to solve the problem. Vern collected 18 bugs that did not have wings.

Topics Reviewed
1. Grams and kilograms; 2. Probabi
3. Polygons; 4. Working backward;
5. Multiplication; 6. Temperature

Investigating the Concept

Modeling Numbers in the Thousands

15–20 MIN Kinesthetic/Logical/Mathematical

Materials *(per group)* Place-Value Blocks or Teaching Tools 2, 3; *(per student)* Place-Value Charts (Teaching Tool 1)

What to Do
- Have students model 1,358.
- **Model a number using at least one thousands block and some hundreds, tens, and ones.**
- **Write the number that represents the model in your place-value chart.**
- Have students take turns modeling and writing numbers for the models.

Thousands Period			Ones
hundreds	tens	ones	hundreds
		1	3

Ongoing Assessment
- Number Sense **If the model has a thousands block, how many in the number?** *(At least 4)*

- **Lesson Organizer**
  - Quick Lesson Overview
  - Professional Development Note
- **Getting Started**
  - Spiral review
  - Investigating the Concept
- **Reaching All Learners**

Instructional Page

Objective To use place value ideas to write multiples of 100 and 1,000 in different ways.

1 Warm Up

Activate Prior Knowledge
Review addition and writing numbers for tallies.

2 Teach

LEARN Emphasize that the place of a digit determines its value in a number. Make sure students can see the relationship between place-value blocks, the expanded form, the standard forms, and the word form of a number.

Example Remind students that when writing numbers in word form not to include *ones* for the ones period.

Ongoing Assessment

Talk About It: Question 3

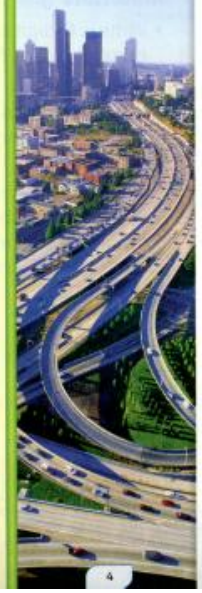
If students say 1,000 for the value of 9 in 129,456,

then remind them that 9 is in the thousands place and there are 9 thousands. The value of 9 in the example is 9 × 1,000, or 9,000.

Lesson 1-1

Key Idea
There are many ways to represent a number.

Vocabulary
- expanded form
- standard form
- word form
- digits
- period



Numbers in the Thousands

WARM UP

What number is shown?

1. 𝍸 |||| 9
2. 30 + 2 32
3. 10 + 10 + 3 23

LEARN

What are some ways to represent numbers in the thousands?

Here are different ways to represent 2,346.

Place-value blocks:

2 thousands 3 hundreds 4 tens 6 ones

Number line:

Expanded form: 2,000 + 300 + 40 + 6
2 thousands + 3 hundreds + 4 tens + 6 ones
(2 × 1,000) + (3 × 100) + (4 × 10) + (6 × 1)

Standard form: 2,346

Word form: two thousand, three hundred forty-six

Digits are the symbols used to write numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

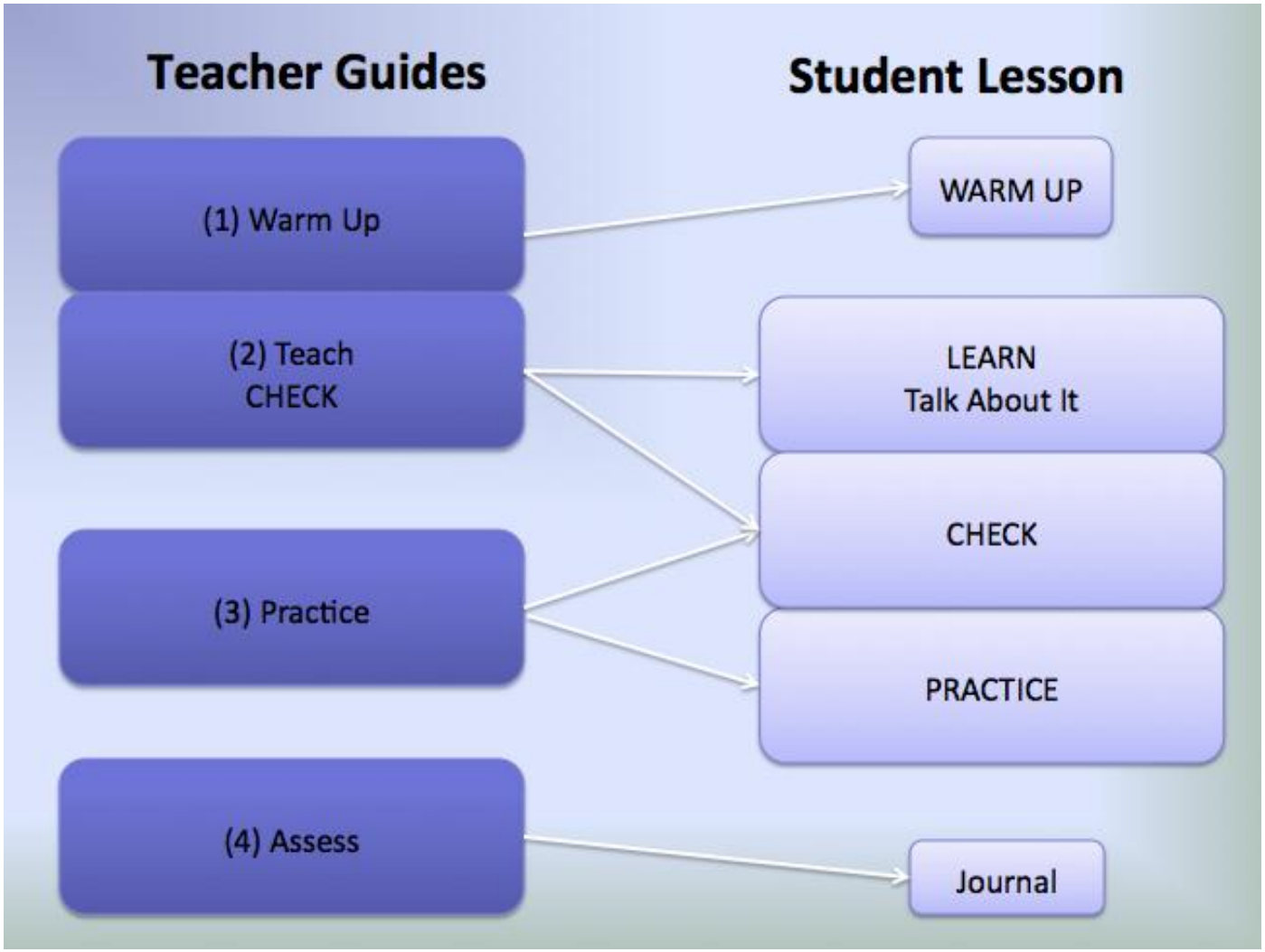
In 2,346, the digit 3 has a **value** of 300 because it is in the hundreds place.

Talk About It

1. Which digit is in the thousands place in 2,346? 2

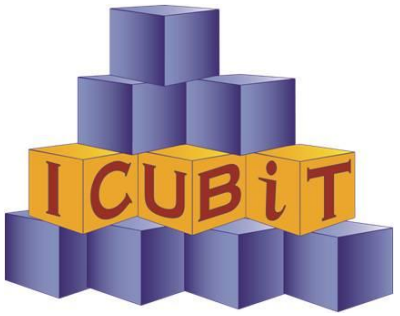
Model Lesson –Common Structure

What does the model lesson include?



Model Lesson – Cognitive Demand

18 Tasks were analyzed



Mem	PWOC	PWC	DM
0 0%	9 50%	9 50%	0 0%

Cognitive Demand – Typical Representations

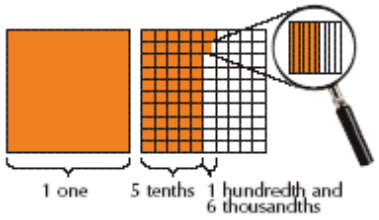
3. 2,048,930

How can you represent decimals?

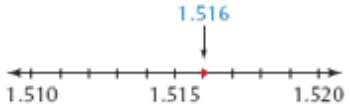
When Trisha went to England, she got 1.516 British pounds for each U.S. dollar.

Here are different ways to represent 1.516.

Grids:



Number line:



Place-value chart:

decimal point

Ones		Tenths	Hundreths	Thousandths
1	.	5	1	6

Expanded form: 1 + 0.5 + 0.01 + 0.006

Standard form: 1.516

Word form: one and five hundred sixteen thousandths

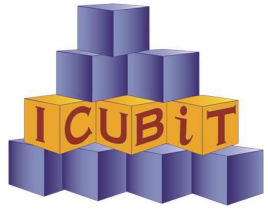
Example

Find 48 ÷ 8.

What You **Think**	What You **Say**	What You **Write**
What number times 8 equals 48?	48 divided by 8 is what number?	48 ÷ 8 = 6 or $8\overline{)48}$ with 6 above
☐ x 8 = 48	*or*	
6 times 8 equals 48. 6 x 8 = 48	How many times does 8 go into 48?	

There are 6 trays in each box.

Guidance for Teachers

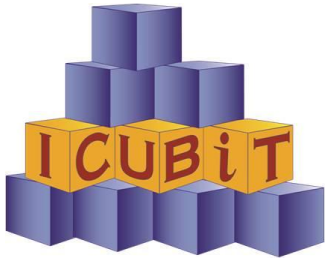


Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
3, 1.5	4	72	62 (86.1%)	0 (0.0%)	7 (9.7%)	3 (4.2%)	1 (1.4%)
3, 3.7	4	79	70 (88.6%)	0 (0.0%)	7 (8.9%)	2 (2.5%)	1 (1.3%)
3, 7.5	4	84	74 (88.1%)	0 (0.0%)	9 (10.7%)	1 (1.2%)	1 (1.2%)
4, 1.3	4	71	58 (81.7%)	0 (0.0%)	12 (16.9%)	1 (1.4%)	1 (1.4%)
4, 3.8	4	75	69 (92.0%)	1 (1.3%)	4 (5.3%)	1 (1.3%)	1 (1.3%)
4, 5.1	4	67	58 (86.6%)	0 (0.0%)	7 (10.5%)	2 (3.0%)	1 (1.5%)
5, 1.3	6	90	79 (87.8%)	2 (2.2%)	6 (6.7%)	3 (3.3%)	3 (3.3%)
5. 4.5	6	110	95 (86.4%)	0(0.0%)	13 (11.8%)	2 (1.8%)	5 (4.6%)
5, 12.1	6	102	83 (81.4%)	1 (1.0%)	10 (9.8%)	8 (7.8%)	4 (3.9%)
Mean Median Range		83.3 79 67 – 110	86.5% 86.6% 81.4-92.0%	0.5% 0.0% 0.0-2.2%	10.0% 9.8% 5.3-16.9%	3.0% 2.5% 1.2-7.8%	2.2% 1.4% 1.2-4.6%

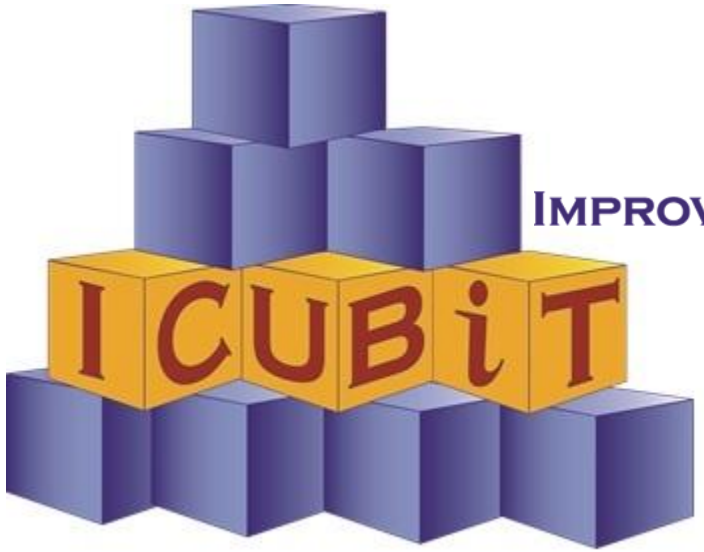
Guidance for Teachers

Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
3, 1.5	4	72	62 (86.1%)	0 (0.0%)	7 (9.7%)	3 (4.2%)	1 (1.4%)
3, 3.7	4	79	70 (88.6%)	0 (0.0%)	7 (8.9%)	2 (2.5%)	1 (1.3%)
3, 7.5	4	84	74 (88.1%)	0 (0.0%)	9 (10.7%)	1 (1.2%)	1 (1.2%)
4, 1.3	4	71	58 (81.7%)	0 (0.0%)	12 (16.9%)	1 (1.4%)	1 (1.4%)
4, 3.8	4	75	69 (92.0%)	1 (1.3%)	4 (5.3%)	1 (1.3%)	1 (1.3%)
4, 5.1	4	67	58 (86.6%)	0 (0.0%)	7 (10.5%)	2 (3.0%)	1 (1.5%)
5, 1.3	6	90	79 (87.8%)	2 (2.2%)	6 (6.7%)	3 (3.3%)	3 (3.3%)
5. 4.5	6	110	95 (86.4%)	0(0.0%)	13 (11.8%)	2 (1.8%)	5 (4.6%)
5, 12.1	6	102	83 (81.4%)	1 (1.0%)	10 (9.8%)	8 (7.8%)	4 (3.9%)
Mean Median Range		83.3 79 67 – 110	86.5% 86.6% 81.4-92.0%	0.5% 0.0% 0.0-2.2%	10.0% 9.8% 5.3-16.9%	3.0% 2.5% 1.2-7.8%	2.2% 1.4% 1.2-4.6%

Demands and Assumptions



Demands on Teacher	Assumed Knowledge
• Follow instructions • Read the lesson • Teach the lesson • Facilitate student practice • (minimal attention given to additional information about math) • (minimal pedagogical supports)	• Teachers know the content • Student page provides the information needed to teach



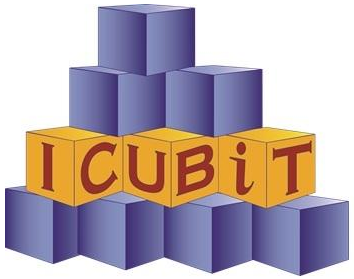
IMPROVING CURRICULUM USE FOR BETTER TEACHING

Analysis of Primary Mathematics (Singapore Math)

Luke Reinke

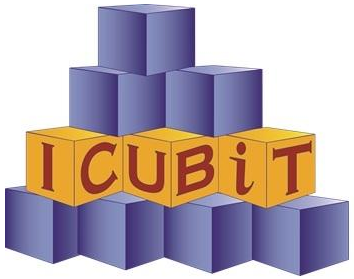
University of Pennsylvania

Primary Mathematics



- Student materials developed by the Ministry of Education (MOE) in Singapore. Teacher's Guide written by authors in the US.
- Student Materials
  - Student textbook
  - Student workbook – consumable
- Teacher Materials
  - Teacher's guide

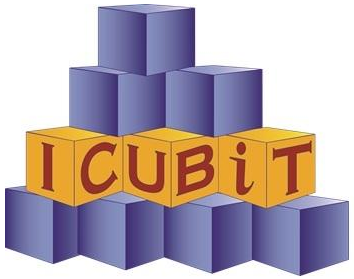
Teachers Guide



Contains:

- Unit Preview
- Chapter Preview explaining connections to prior lessons and mathematical representations (not analyzed)
- Lessons

Instructional Page

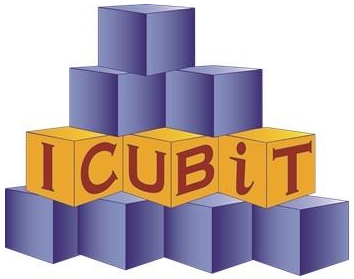


Lesson 1.1a **Billions**

Objectives	**California Standards**
• Write very large numbers in words, standard form and expanded form. • Understand place value for very large numbers. • Compare very large numbers.	**NS 1.1**: Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers. **MR 1.2**: Determine when and how to break a problem into simpler parts. **MR 2.2**: Apply strategies and results from simpler problems to more complex problems. **MR 2.3**: Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
Materials • Place value strips	**Vocabulary/Phrases** Billion Period

Teaching Strategies		
Introduce billions	Write the number '476,218,590' on the board and ask students to read the number. (four hundred seventy-six million, two hundred eighteen thousand, five hundred ninety)	476,218,590
	Write a '3' in front of the earlier number to form '3,476,218,590'. Ask students if they know how to read it.	3,476,218,590
	Tell student that the additional digit is in the billions place. The number now reads "three billion, four hundred seventy-six million, two hundred eighteen thousand, five hundred ninety".	
	Tell students that 1 billion is equal to 1 thousand millions.	1 billion = 1000 millions
	Tell students that we use commas to help us write very large numbers in both words and figures. Groups of 3 digits in large numbers are called periods and are separated by commas. They are grouped into ones, thousands, millions, billions, and so on. Have students look at the place value table on **Textbook p. 8** for an illustration of the periods in very large numbers.	

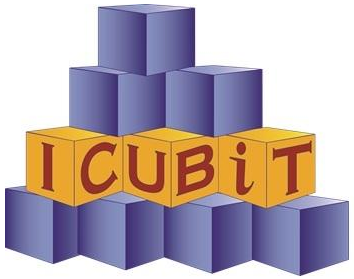
Model Lesson- Common Structure



What does the model lesson include?

- Demonstration
- Student Assignment
- Game or Activity
- Practice from workbook

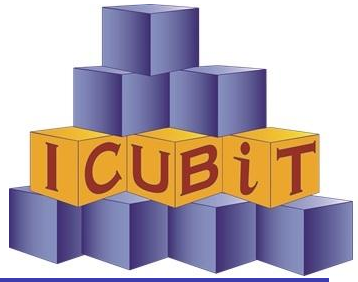
Model Lesson – Cognitive Demand



21 Tasks were analyzed

Mem	PWOC	PWC	DM
0 0%	10 47.6%	9 42.9%	2 9.5%

Model Lesson - Roles



Teachers' Role	Students' Role
• Follows the instructions in the teacher's guide to model the procedures	• Listens to teacher's presentation
• Ask questions suggested by the teacher's guide	• Answer teacher questions
• Assign task to students	• Following the presentation, students are to practice the procedures that were modeled by the teacher

Guidance for Teachers

Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
Gr 3, 1.1	5	77	71 (92.2%)	0 (0.0%)	5 (6.5%)	1 (1.3%)	0 (0.0%)
Gr 3, 2.7	3	56	53 (94.6%)	0 (0.0%)	1 (1.8%)	2 (3.6%)	1 (1.8%)
Gr 3, 4.3	4	77	73 (94.8%)	1 (1.3%)	2 (2.6%)	1 (1.3%)	2 (2.6%)
Gr 4, 1.1	4	64	49 (76.6%)	2 (3.1%)	6 (9.4%)	7 (10.9%)	1 (1.6%)
Gr 4, 1.5	3	30	24 (80.0%)	0 (0.0%)	2 (6.7%)	4 (13.3%)	0 (0.0%)
Gr 4, 2.1	5	62	49 (79.0%)	2 (3.2%)	6 (9.7%)	5 (8.1%)	1 (1.6%)
Gr 5, 1.1	3	43	41 (95.3%)	0 (0.0%)	0 (0.0%)	2 (4.7%)	0 (0.0%)
Gr 5, 1.4	5	90	80 (88.9%)	1 (1.1%)	7 (7.8%)	2 (2.2%)	0 (0.0%)
Gr 5, 13.1	3	39	35 (89.7%)	0 (0.0%)	1 (2.6%)	3 (7.7%)	0 (0.0%)
Mean Median Range		59.8	87.91 89.7 76.6-95.3	0.97 0.0 0.0-3.2	5.22 6.5 0.0-9.7	5.90 4.7 1.3-13.3	0.84 0.0 0.-2.6

Guidance for Teachers

Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
Gr 3, 1.1	5	77	71 (92.2%)	0 (0.0%)	5 (6.5%)	1 (1.3%)	0 (0.0%)
Gr 3, 2.7	3	56	53 (94.6%)	0 (0.0%)	1 (1.8%)	2 (3.6%)	1 (1.8%)
Gr 3, 4.3	4	77	73 (94.8%)	1 (1.3%)	2 (2.6%)	1 (1.3%)	2 (2.6%)
Gr 4, 1.1	4	64	49 (76.6%)	2 (3.1%)	6 (9.4%)	7 (10.9%)	1 (1.6%)
Gr 4, 1.5	3	30	24 (80.0%)	0 (0.0%)	2 (6.7%)	4 (13.3%)	0 (0.0%)
Gr 4, 2.1	5	62	49 (79.0%)	2 (3.2%)	6 (9.7%)	5 (8.1%)	1 (1.6%)
Gr 5, 1.1	3	43	41 (95.3%)	0 (0.0%)	0 (0.0%)	2 (4.7%)	0 (0.0%)
Gr 5, 1.4	5	90	80 (88.9%)	1 (1.1%)	7 (7.8%)	2 (2.2%)	0 (0.0%)
Gr 5, 13.1	3	39	35 (89.7%)	0 (0.0%)	1 (2.6%)	3 (7.7%)	0 (0.0%)
Mean		59.8	87.91	0.97	5.22	5.90	0.84
Median			89.7	0.0	6.5	4.7	0.0
Range			76.6-95.3	0.0-3.2	0.0-9.7	1.3-13.3	0.-2.6

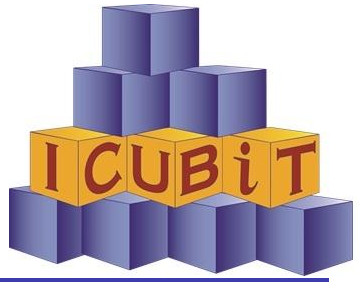
Guidance for Teachers

Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
Gr 3, 1.1	5	77	71 (92.2%)	0 (0.0%)	5 (6.5%)	1 (1.3%)	0 (0.0%)
Gr 3, 2.7	3	56	53 (94.6%)	0 (0.0%)	1 (1.8%)	2 (3.6%)	1 (1.8%)
Gr 3, 4.3	4	77	73 (94.8%)	1 (1.3%)	2 (2.6%)	1 (1.3%)	2 (2.6%)
Gr 4, 1.1	4	64	49 (76.6%)	2 (3.1%)	6 (9.4%)	7 (10.9%)	1 (1.6%)
Gr 4, 1.5	3	30	24 (80.0%)	0 (0.0%)	2 (6.7%)	4 (13.3%)	0 (0.0%)
Gr 4, 2.1	5	62	49 (79.0%)	2 (3.2%)	6 (9.7%)	5 (8.1%)	1 (1.6%)
Gr 5, 1.1	3	43	41 (95.3%)	0 (0.0%)	0 (0.0%)	2 (4.7%)	0 (0.0%)
Gr 5, 1.4	5	90	80 (88.9%)	1 (1.1%)	7 (7.8%)	2 (2.2%)	0 (0.0%)
Gr 5, 13.1	3	39	35 (89.7%)	0 (0.0%)	1 (2.6%)	3 (7.7%)	0 (0.0%)
Mean Median Range		59.8	87.91 89.7 76.6-95.3	0.97 0.0 0.0-3.2	5.22 6.5 0.0-9.7	5.90 4.7 1.3-13.3	0.84 0.0 0.-2.6

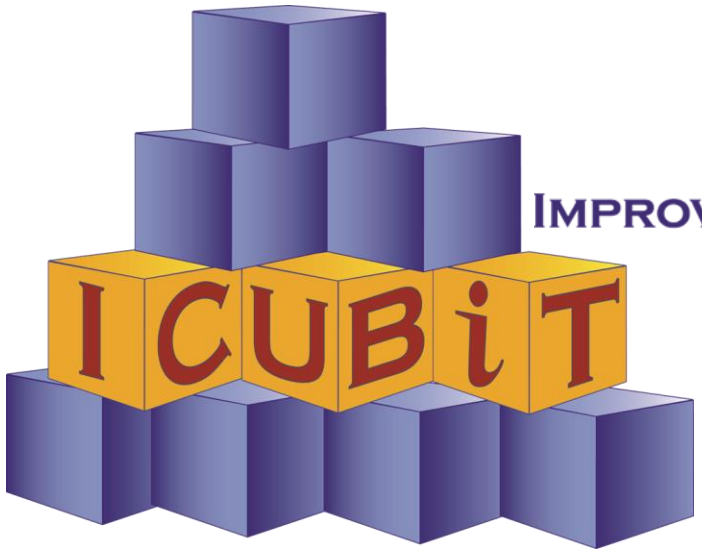
Guidance for Teachers

Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
Gr 3, 1.1	5	77	71 (92.2%)	0 (0.0%)	5 (6.5%)	1 (1.3%)	0 (0.0%)
Gr 3, 2.7	3	56	53 (94.6%)	0 (0.0%)	1 (1.8%)	2 (3.6%)	1 (1.8%)
Gr 3, 4.3	4	77	73 (94.8%)	1 (1.3%)	2 (2.6%)	1 (1.3%)	2 (2.6%)
Gr 4, 1.1	4	64	49 (76.6%)	2 (3.1%)	6 (9.4%)	7 (10.9%)	1 (1.6%)
Gr 4, 1.5	3	30	24 (80.0%)	0 (0.0%)	2 (6.7%)	4 (13.3%)	0 (0.0%)
Gr 4, 2.1	5	62	49 (79.0%)	2 (3.2%)	6 (9.7%)	5 (8.1%)	1 (1.6%)
Gr 5, 1.1	3	43	41 (95.3%)	0 (0.0%)	0 (0.0%)	2 (4.7%)	0 (0.0%)
Gr 5, 1.4	5	90	80 (88.9%)	1 (1.1%)	7 (7.8%)	2 (2.2%)	0 (0.0%)
Gr 5, 13.1	3	39	35 (89.7%)	0 (0.0%)	1 (2.6%)	3 (7.7%)	0 (0.0%)
Mean		59.8	87.91	0.97	5.22	5.90	0.84
Median			89.7	0.0	6.5	4.7	0.0
Range			76.6-95.3	0.0-3.2	0.0-9.7	1.3-13.3	0.-2.6

Demands and Assumptions



Demands on Teacher	Assumed Knowledge
• Must be able to manage the detailed instructions while presenting in an engaging way	• Knowledge of content and students – How to respond to student questions and misconceptions – How to differentiate or plan for their specific context



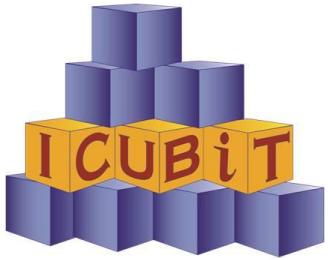
IMPROVING CURRICULUM USE FOR BETTER TEACHING

Analysis of Math Trailblazers

Ok-Kyeong Kim

Western Michigan University

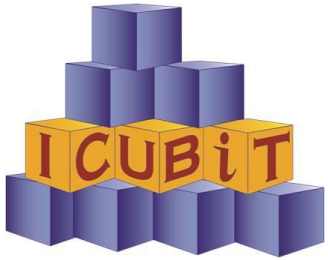
Math Trailblazers (3rd Ed)



- Developed by Teaching Integrated Mathematics and Science Project (TIMS) Project, University of Illinois at Chicago

  (K-5, Standards-based, NSF-funded)
- Organization: 16-20 units per grade, 5-9 lessons per unit, 1-5 sessions (mostly 1-2) per lesson
- Materials for students: Student Guide, Discovery Assignment Book (grades 3-5), and Adventure Book

Math Trailblazers



- **Materials for teachers:**

  - Unit Resource Guides: for day-to-day teaching

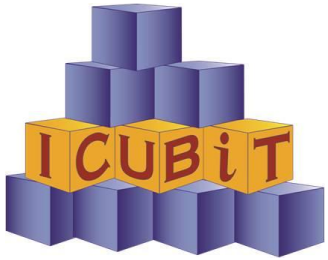
  - Facts Resource Guide: daily practice problems (DPP)

  - Teacher Implementation Guide:

  philosophy of the curriculum
  overview of each unit
  assessment
  math facts and practice
  ***TIMS* tutors**

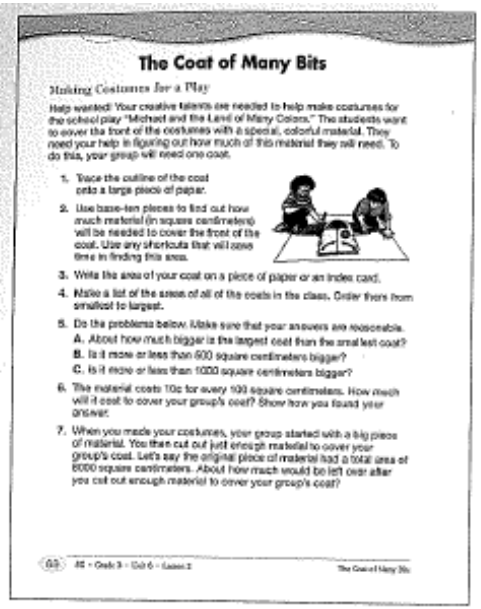
  - Teacher Resource CD

Unit Resource Guides



Each unit resource guide includes:

- unit outline and pacing suggestions
- background information about the main topics or mathematical ideas of the unit
- assessment indicators
- daily practice problems of the unit
- a letter to students' parents



The Coat of Many Bits

Making Costumes for a Play

Help wanted! Your creative talents are needed to help make costumes for the school play "Michael and the Land of Many Colors." The students want to cover the front of the costumes with a special, colorful material. They need your help in figuring out how much of this material they will need. To do this, your group will need one coat.

1. Trace the outline of the coat onto a large piece of paper.
2. Use base-ten pieces to find out how much material (in square centimeters) will be needed to cover the front of the coat. Use any shortcuts that will save time in finding this area.
3. Write the area of your coat on a piece of paper or an index card.
4. Make a list of the areas of all of the coats in the class. Order them from smallest to largest.
5. Do the problems below. Make sure that your answers are reasonable.
   A. About how much bigger is the largest coat than the smallest coat?
   B. Is it more or less than 500 square centimeters bigger?
   C. Is it more or less than 1000 square centimeters bigger?
6. The material costs 10¢ for every 100 square centimeters. How much will it cost to cover your group's coat? Show how you found your answer.
7. When you made your costumes, your group started with a big piece of material. You then cut out just enough material to cover your group's coat. Let's say the original piece of material had a total area of 6000 square centimeters. About how much would be left over after you cut out enough material to cover your group's coat?

Student Guide - page 66 *(Answers on p. 40)*

Journal Prompt

Why are bits, skinnies, and flats appropriate for measuring area but packs are not?

Before the Activity

Cut sheets of butcher paper into pieces large enough for students to lay a coat or jacket on and trace the entire outline. About 2 yards by 1 yard is usually adequate.

Teaching the Activity

The Coat of Many Bits Activity Page in the *Student Guide* describes the context for the lesson. A group of students are producing a play entitled "Michael and the Land of Many Colors." Your students will assist with the production by helping to make the costumes. Other aspects of the play become the topic of mathematical explorations in a later activity.

The activity page explains that the front of the costumes will be covered with a fancy, colorful material. To do this, students first trace the outline of a coat on a large sheet of paper. This gives them a picture (or model) of the coat. They use this picture to find out how much material they need to cover the front.

In Unit 5 students measured area by tracing shapes on centimeter grids and counting square centimeters. In this activity they count square centimeters using base-ten pieces. Ask students how they could use base-ten pieces to estimate the area of the front of their coats. If necessary, point out that the different sizes of base-ten pieces cover a different number of square centimeters—one bit covers one square centimeter—therefore a skinny covers ten square centimeters, and a flat covers 100 square centimeters.

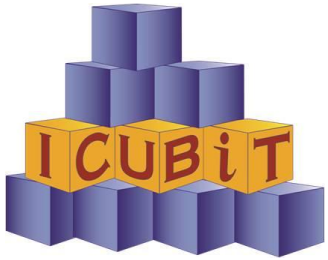
Model the activity by tracing the outline of a student's coat on the board (with the help of a couple of students).

There are several practical points to discuss, such as:

- Each group will only find the area of one coat; this will be the group's coat.
- The coats should be zipped (or snapped or buttoned) and the sleeves should be extended.
- Hoods of coats should not be included since only the fronts of the coats will be covered with fancy material.
- The coats should be held flat against the surfaces they are traced on.
- Do not use markers when tracing because they may stain the coats.

Sample instructional page

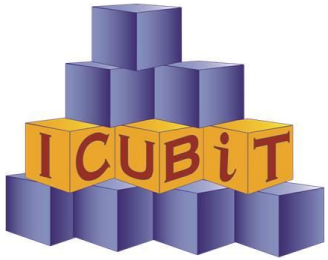
Model Lesson – Common Structure



What does the model lesson include?

- 1-3 main activities/tasks in whole-group, individual, pair/small-group settings
- math facts, homework and practice (DPP)
- assessment
- extension occasionally

Model Lesson – Cognitive Demand

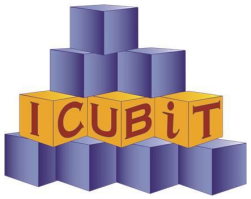


- **Nature of mathematical tasks:**

  15 main tasks in 9 lessons analyzed in number and operations and algebra strands

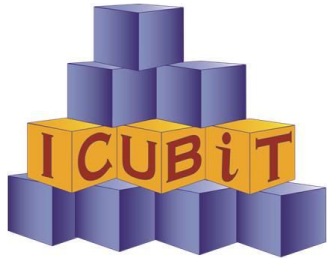
Mem	PWOC	PWC	DM
-	-	11 (73%) 2 (13%) PWC/DM	2 (13%)

Model Lesson – Roles



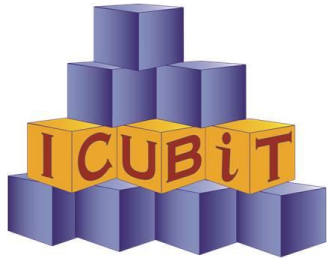
Teacher Role	Student Role
• Provide critical facts to organize discussion and ask students to justify/explain their thinking • Promote diverse and analytic thinking (e.g., comparing various computation methods) • Ensure students to develop computational and problem solving skills on a daily basis	• Collect and organize data • Find and discuss patterns from data • Develop strategies for number problems • Compare various methods and choose strategies • Communicate their strategies and thinking both verbally and in writing

Guidance for Teachers



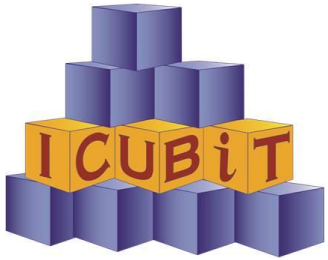
Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
G3, 6.1	4 2/3	74	64 (86.5%)	9 (12.2%)	2 (2.7%)	0 (0%)	5 (6.8%)
G3, 6.2	6	91	64 (70.3%)	15 (16.5%)	12 (13.2%)	0 (0%)	6 (6.6%)
G3, 6.3	9	130	85 (65.4%)	17 (13.1%)	14 (10.8%)	14 (10.8%)	17 (13.1%)
G4, 3.1	10	176	90 (51.1%)	37 (21.0%)	30 (17.0%)	21 (11.9%)	2 (1.1%)
G4, 6.2	9	148	94 (63.5%)	17 (11.5%)	21 (14.2%)	24 (16.2%)	8 (5.4%)
G4, 15.4	7	91	55 (60.4%)	21 (23.1%)	3 (3.3%)	16 (17.6%)	5 (5.5%)
G5, 2.3	11	270	164 (60.7%)	27 (10.0%)	70 (25.9%)	17 (6.3%)	10 (3.7%)
G5, 9.3	7	78	55 (70.5%)	8 (10.3%)	11 (14.1%)	4 (5.1%)	1 (1.3%)
G5, 11.2	6 2/3	99	60 (60.6%)	8 (8.0%)	17 (17.2%)	26 (26.3%)	9 (9.1%)
Mean		128.6	65.5%	14.0%	13.2%	10.5%	5.9%%
Median		99	63.5%	12.2%	14.1%	10.8%	5.5%
Range		74-270	51.1-86.5%	8.1-23.1%	2.7-25.9%	0-26.3%	1.2-13.1%

Guidance for Teachers



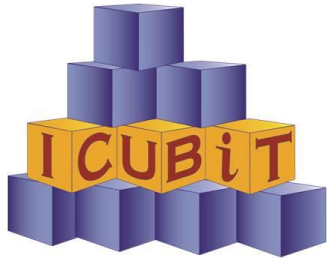
Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
G3, 6.1	4 2/3	74	64 (86.5%)	9 (12.2%)	2 (2.7%)	0 (0%)	5 (6.8%)
G3, 6.2	6	91	64 (70.3%)	15 (16.5%)	12 (13.2%)	0 (0%)	6 (6.6%)
G3, 6.3	9	130	85 (65.4%)	17 (13.1%)	14 (10.8%)	14 (10.8%)	17 (13.1%)
G4, 3.1	10	176	90 (51.1%)	37 (21.0%)	30 (17.0%)	21 (11.9%)	2 (1.1%)
G4, 6.2	9	148	94 (63.5%)	17 (11.5%)	21 (14.2%)	24 (16.2%)	8 (5.4%)
G4, 15.4	7	91	55 (60.4%)	21 (23.1%)	3 (3.3%)	16 (17.6%)	5 (5.5%)
G5, 2.3	11	270	164 (60.7%)	27 (10.0%)	70 (25.9%)	17 (6.3%)	10 (3.7%)
G5, 9.3	7	78	55 (70.5%)	8 (10.3%)	11 (14.1%)	4 (5.1%)	1 (1.3%)
G5, 11.2	6 2/3	99	60 (60.6%)	8 (8.0%)	17 (17.2%)	26 (26.3%)	9 (9.1%)
Mean		128.6	65.5%	14.0%	13.2%	10.5%	5.9%%
Median		99	63.5%	12.2%	14.1%	10.8%	5.5%
Range		74-270	51.1-86.5%	8.1-23.1%	2.7-25.9%	0-26.3%	1.2-13.1%

Explain rationale – example



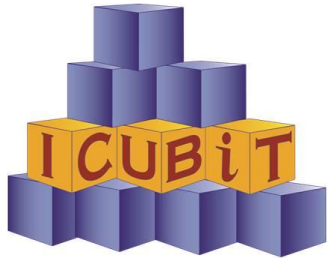
- **Fact families are introduced so students can use multiplication facts to learn related division facts. They use flash cards to assess their fluency with multiplication facts for the fives and tens. (Lesson Overview, p. 24)**
- **In Units 3-7, students use the *Triangle Flash Cards* and the *Facts I know* charts only with the multiplication facts. They will build strategies for the division facts in Units 3-8 and use the *Triangle Flash Cards* to develop fluency with division facts in Units 9-16. Reviewing the multiplication facts will facilitate their work with the division facts. (Content Note, p. 28)**
- **Having students draw pictures reinforces their understanding of the concepts represented in the number sentences. (Teaching the Activity, p. 29).**

Guidance for Teachers



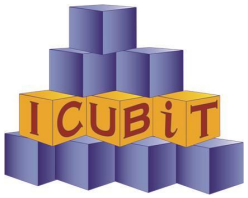
Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
G3, 6.1	4 2/3	74	64 (86.5%)	9 (12.2%)	2 (2.7%)	0 (0%)	5 (6.8%)
G3, 6.2	6	91	64 (70.3%)	15 (16.5%)	12 (13.2%)	0 (0%)	6 (6.6%)
G3, 6.3	9	130	85 (65.4%)	17 (13.1%)	14 (10.8%)	14 (10.8%)	17 (13.1%)
G4, 3.1	10	176	90 (51.1%)	37 (21.0%)	30 (17.0%)	21 (11.9%)	2 (1.1%)
G4, 6.2	9	148	94 (63.5%)	17 (11.5%)	21 (14.2%)	24 (16.2%)	8 (5.4%)
G4, 15.4	7	91	55 (60.4%)	21 (23.1%)	3 (3.3%)	16 (17.6%)	5 (5.5%)
G5, 2.3	11	270	164 (60.7%)	27 (10.0%)	70 (25.9%)	17 (6.3%)	10 (3.7%)
G5, 9.3	7	78	55 (70.5%)	8 (10.3%)	11 (14.1%)	4 (5.1%)	1 (1.3%)
G5, 11.2	6 2/3	99	60 (60.6%)	8 (8.0%)	17 (17.2%)	26 (26.3%)	9 (9.1%)
Mean		128.6	65.5%	14.0%	13.2%	10.5%	5.9%%
Median		99	63.5%	12.2%	14.1%	10.8%	5.5%
Range		74-270	51.1-86.5%	8.1-23.1%	2.7-25.9%	0-26.3%	1.2-13.1%

Guidance for Teachers



Lesson	Pages per Lesson	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student Thinking	Explaining math	Support Decision Making
G3, 6.1	4 2/3	74	64 (86.5%)	9 (12.2%)	2 (2.7%)	0 (0%)	5 (6.8%)
G3, 6.2	6	91	64 (70.3%)	15 (16.5%)	12 (13.2%)	0 (0%)	6 (6.6%)
G3, 6.3	9	130	85 (65.4%)	17 (13.1%)	14 (10.8%)	14 (10.8%)	17 (13.1%)
G4, 3.1	10	176	90 (51.1%)	37 (21.0%)	30 (17.0%)	21 (11.9%)	2 (1.1%)
G4, 6.2	9	148	94 (63.5%)	17 (11.5%)	21 (14.2%)	24 (16.2%)	8 (5.4%)
G4, 15.4	7	91	55 (60.4%)	21 (23.1%)	3 (3.3%)	16 (17.6%)	5 (5.5%)
G5, 2.3	11	270	164 (60.7%)	27 (10.0%)	70 (25.9%)	17 (6.3%)	10 (3.7%)
G5, 9.3	7	78	55 (70.5%)	8 (10.3%)	11 (14.1%)	4 (5.1%)	1 (1.3%)
G5, 11.2	6 2/3	99	60 (60.6%)	8 (8.0%)	17 (17.2%)	26 (26.3%)	9 (9.1%)
Mean		128.6	65.5%	14.0%	13.2%	10.5%	5.9%%
Median		99	63.5%	12.2%	14.1%	10.8%	5.5%
Range		74-270	51.1-86.5%	8.1-23.1%	2.7-25.9%	0-26.3%	1.2-13.1%

Demands and Assumptions



Demands on Teacher

- Heavy reading in terms of content and guidance
- Careful examination and in-depth understanding of the mathematics
- Balance between understanding and skills
- Use of tools for instruction (calculators, manipulatives, representations)

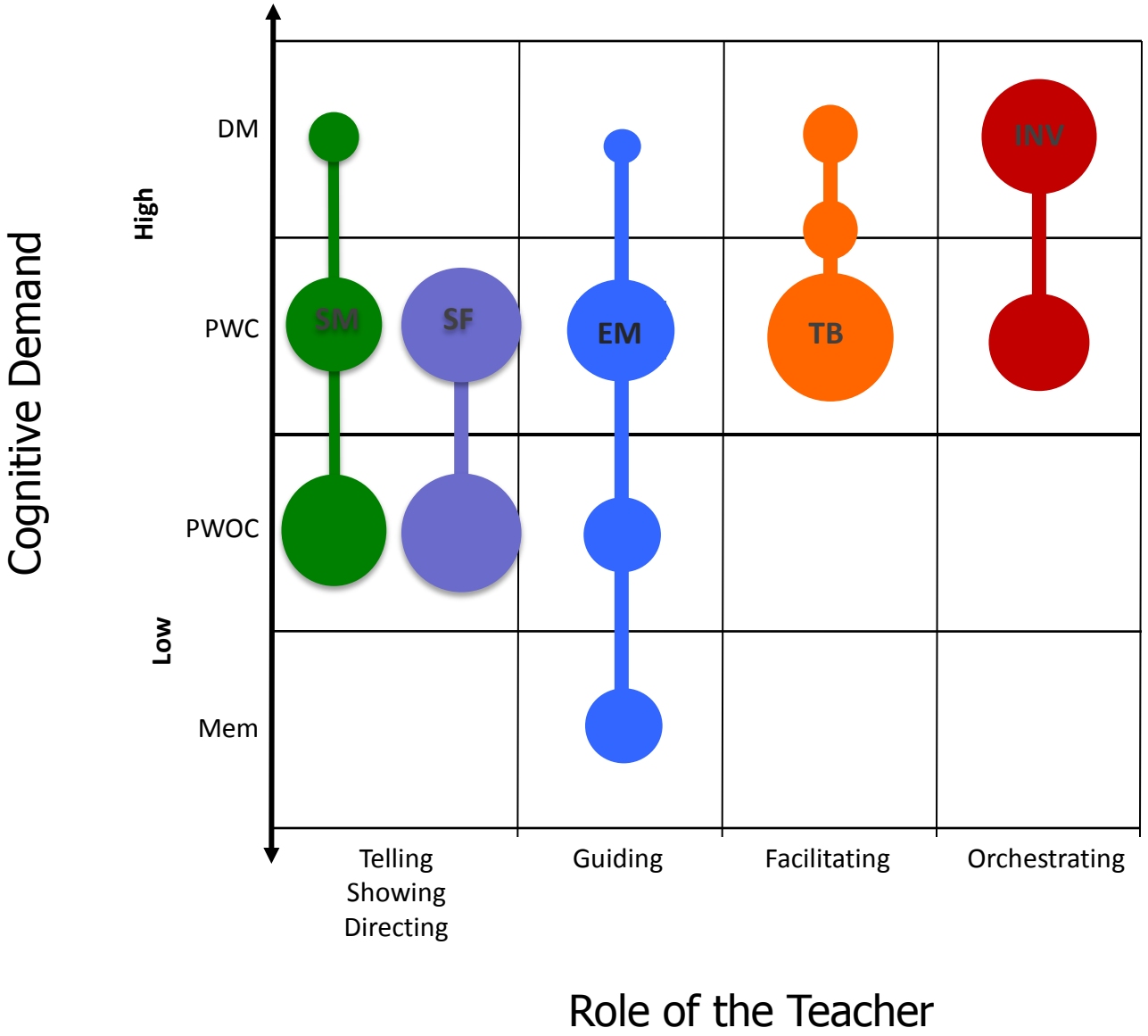
Assumed Knowledge

- Sophisticated and advanced knowledge of mathematics
- Knowledge of reform recommendations (e.g., NCTM Standards)

Cognitive Demand

n per curriculum	Memorization	PWOC	PWC	Doing Math
EM n=18	4 (22%)	4 (22%)	9 (50%)	1 (6%)
INV n=11	-	-	5 (45%)	6 (55%)
SF n=18	-	9 (50%)	9 (50%)	-
SM n=21	-	10 (48%)	9 (43%)	2 (9%)
TB n=15	-	-	11 (73%) 2 (13%) PWC/DM	2 (13%)

Role of the Teacher



		Percent of Total Number of Sentences/Phrases Devoted to. . .				
	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6 68.2-87.6	8.3 5.4-13.9	7.5 0-12.9	5.6 0.0-18.9	7.5 3.4-11.9
INV	114.8	74.3 61.8-81.4	6.8 1.5-12.3	12.8 7.8-23.3	3.9 0.0-10.8	2.2 0.0-5.5
SF	83.3	86.5 81.4-92.0	0.5 0.0-2.2	10.0 5.3-16.9	3.0 1.2-7.8	2.2 1.2-4.6
SM	59.8	87.91 76.6-95.3	1.0 0.0-3.2	5.2 0.0-9.7	5.9 1.3-13.3	0.8 0.0-2.6
TB	128.6	65.5 51.1-86.5	14.0 8.1-23.1	13.2 2.7-25.9	10.5 0.0-26.3	5.8 1.2-13.1

		Percent of Total Number of Sentences/Phrases Devoted to. . .				
	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6 68.2-87.6	8.3 5.4-13.9	7.5 0-12.9	5.6 0.0-18.9	7.5 3.4-11.9
INV	114.8	74.3 61.8-81.4	6.8 1.5-12.3	12.8 7.8-23.3	3.9 0.0-10.8	2.2 0.0-5.5
SF	83.3	86.5 81.4-92.0	0.5 0.0-2.2	10.0 5.3-16.9	3.0 1.2-7.8	2.2 1.2-4.6
SM	59.8	87.91 76.6-95.3	1.0 0.0-3.2	5.2 0.0-9.7	5.9 1.3-13.3	0.8 0.0-2.6
TB	128.6	65.5 51.1-86.5	14.0 8.1-23.1	13.2 2.7-25.9	10.5 0.0-26.3	5.8 1.2-13.1

		Percent of Total Number of Sentences/Phrases Devoted to. . .				
	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6 68.2-87.6	8.3 5.4-13.9	7.5 0-12.9	5.6 0.0-18.9	7.5 3.4-11.9
INV	114.8	74.3 61.8-81.4	6.8 1.5-12.3	12.8 7.8-23.3	3.9 0.0-10.8	2.2 0.0-5.5
SF	83.3	86.5 81.4-92.0	0.5 0.0-2.2	10.0 5.3-16.9	3.0 1.2-7.8	2.2 1.2-4.6
SM	59.8	87.91 76.6-95.3	1.0 0.0-3.2	5.2 0.0-9.7	5.9 1.3-13.3	0.8 0.0-2.6
TB	128.6	65.5 51.1-86.5	14.0 8.1-23.1	13.2 2.7-25.9	10.5 0.0-26.3	5.8 1.2-13.1

		Percent of Total Number of Sentences/Phrases Devoted to. . .				
	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6 68.2-87.6	8.3 5.4-13.9	7.5 0-12.9	5.6 0.0-18.9	7.5 3.4-11.9
INV	114.8	74.3 61.8-81.4	6.8 1.5-12.3	12.8 7.8-23.3	3.9 0.0-10.8	2.2 0.0-5.5
SF	83.3	86.5 81.4-92.0	0.5 0.0-2.2	10.0 5.3-16.9	3.0 1.2-7.8	2.2 1.2-4.6
SM	59.8	87.91 76.6-95.3	1.0 0.0-3.2	5.2 0.0-9.7	5.9 1.3-13.3	0.8 0.0-2.6
TB	128.6	65.5 51.1-86.5	14.0 8.1-23.1	13.2 2.7-25.9	10.5 0.0-26.3	5.8 1.2-13.1

		Percent of Total Number of Sentences/Phrases Devoted to. . .				
	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6 68.2-87.6	8.3 5.4-13.9	7.5 0-12.9	5.6 0.0-18.9	7.5 3.4-11.9
INV	114.8	74.3 61.8-81.4	6.8 1.5-12.3	12.8 7.8-23.3	3.9 0.0-10.8	2.2 0.0-5.5
SF	83.3	86.5 81.4-92.0	0.5 0.0-2.2	10.0 5.3-16.9	3.0 1.2-7.8	2.2 1.2-4.6
SM	59.8	87.91 76.6-95.3	1.0 0.0-3.2	5.2 0.0-9.7	5.9 1.3-13.3	0.8 0.0-2.6
TB	128.6	65.5 51.1-86.5	14.0 8.1-23.1	13.2 2.7-25.9	10.5 0.0-26.3	5.8 1.2-13.1

Percent of Total Number of Sentences/Phrases Devoted to. . .

	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6 68.2-87.6	8.3 5.4-13.9	7.5 0-12.9	5.6 0.0-18.9	7.5 3.4-11.9
INV	114.8	74.3 61.8-81.4	6.8 1.5-12.3	12.8 7.8-23.3	3.9 0.0-10.8	2.2 0.0-5.5
SF	83.3	86.5 81.4-92.0	0.5 0.0-2.2	10.0 5.3-16.9	3.0 1.2-7.8	2.2 1.2-4.6
SM	59.8	87.91 76.6-95.3	1.0 0.0-3.2	5.2 0.0-9.7	5.9 1.3-13.3	0.8 0.0-2.6
TB	128.6	65.5 51.1-86.5	14.0 8.1-23.1	13.2 2.7-25.9	10.5 0.0-26.3	5.8 1.2-13.1